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Critical dissipation rates in density stratified turbulence

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The estimation of a critical dissipation rate, ϵ_c, below which vertical transport $\overline{\rho w}$ is completely suppressed, is examined in the context of laboratory experiments on shear-free, decaying, and stably stratified grid-generated turbulence. It is shown how the use of a criterion based on $\overline{\rho w}=0$ in these transient flows is not appropriate for obtaining a universal value for the critical dissipation rate expressed nondimensionally as $\epsilon_c/\nu N^2$. It remains unclear how any universal value of ϵ_c may be inferred from these laboratory experiments. © *1995 American Institute of Physics.*

I. INTRODUCTION

Stable density stratification (e.g., arising from variations of temperature, salinity, or suspended solids) is common in the ocean. Yet, there is a lack of understanding of transport processes in density stratified flows. In particular, "there seems to be no clear answers yet to the problems of understanding and parameterizing vertical mixing in the ocean" (Garrett[1]).

Estimates of rates of vertical transport in a stratified ocean comprise a central question in the analysis/debate on climate change. Turner[2] argued that the rate of change of potential energy of the fluid column is dependent upon the rate of supply of turbulent kinetic energy, so that a major effect of a stable density stratification is to attenuate rates of vertical transport. Thus, a question that has been a focus of attention is, can sufficiently large values of stable density gradients effectively extinguish vertical transport? Alternatively expressed, what is the minimum value of the rate of supply of turbulent kinetic energy necessary to overcome the buoyancy forces in stably stratified fluid column, so as to result in vertical transport and therefore an increase in potential energy? In what follows, the results of experiments designed to address this question by measuring, directly, the vertical transport $\overline{\rho w}$, and the turbulent velocity field in a stably stratified flow are described.

Stable stratification influences a turbulent flow field when turbulent kinetic energy is depleted by work against gravity, so that relative to an otherwise identical configuration without density stratification velocity variances may be expected to diminish. A convenient way to estimate whether there is vertical transport is in terms of the dissipation rate ϵ of the flow expressed as a nondimensional quantity $\epsilon/\nu N^2$, where ν is the kinematic viscosity and N the buoyancy frequency $N=[-g/\bar{\rho}(\partial\bar{\rho}/\partial z)]^{1/2}$ (Gibson[3] and Gregg[4]). The physics behind this choice of parameter can be understood by considering the inertial-buoyancy force balance of a turbulent stably stratified flow. The inertial-buoyancy force balance for motions of velocity scale u' and the length scale l is expressed by the turbulent Froude number $F_R=u'/Nl$. Equating the length scale l with a dissipation length scale $l=u'^3/\epsilon$ (where ϵ is the turbulent kinetic energy dissipation rate), it is possible to derive a length scale at which a balance between buoyancy and inertial forces arises (i.e., $F_R=1$)—the Ozmidov scale $l_0=(\epsilon/N^3)^{1/2}$. A comparison between the Ozmidov scale and the smallest scales of turbulent motion, as given by the Kolmogorov length scale $l_K=(\nu^3/\epsilon)^{1/4}$ is easily seen to be related to the parameter $\epsilon/\nu N^2$. These arguments suggest that there may be a critical dissipation rate ϵ_c below which all scales of turbulent motion will be suppressed by buoyancy forces, and therefore that the flow could not sustain a turbulent flux $\overline{\rho w}$. Note that we make a distinction between the critical dissipation rate ϵ_c and ϵ_0, the dissipation rate at the initial location where the flux $\overline{\rho w}=0$ in developing grid-generated turbulence, as discussed by previous authors. The reason for this distinction will become clear as our discussion develops.

Another interpretation of $F_R=u'/Nl$ is in terms of a ratio of time scales, since N has dimensions of $(\text{time})^{-1}$, that is, $F_R=u'/Nl=(Nt)^{-1}$, where t is the turbulent time scale. When F_R is large or Nt is small, the turbulence will be largely unaffected by stratification and so evolve approximately, as in the case without density stratification. That the nature and evolution of turbulence in a stable density stratified flow is well described in terms of the value of the parameter Nt has been experimentally verified, particularly in recent papers with direct measurements of flux $\overline{\rho w}$, by Stillinger *et al.*,[5] Itsweire *et al.*,[6] Lienhard and Van Atta,[7] Yoon and Warhaft,[8] and Jayesh *et al.*[9] A consensus among these experimental studies is that vertical transport $\overline{\rho w}$ vanishes at $Nt\sim O(1)$. There remains controversy about the nature of turbulent stratified flow for values of $Nt\gg 1$. There is disagreement as to whether the flow is best described as a mixture between turbulence and internal waves, or interacting internal waves or fossil turbulence (Bretherton,[10] Dickey and Mellor,[11] and Gibson[12]).

Gibson[3] from theoretical arguments suggested a critical dissipation rate $\epsilon_c/\nu N^2=30$ necessary to support a vertical flux. Attempts have been made to verify this by laboratory experiments. The approach taken has been to use shear-free, decaying grid-generated turbulence in a uniformly stratified fluid. In such flows the magnitude of the flux F grows initially, then subsequently diminishes to zero and oscillates with small values of F and for large values of Nt. The initial zero crossing of the flux curve has been used to define the point where critical dissipation rates may be characterized. Values of the critical dissipation rate deduced in this way vary by a factor of about 3. Stillinger *et al.*[5] found $\epsilon_c/\nu N^2=25$. Itsweire *et al.*[6] found $\epsilon_c/\nu N^2=21$ and 15 for grid mesh sizes of 3.81 and 1.9 cm, respectively, whereas

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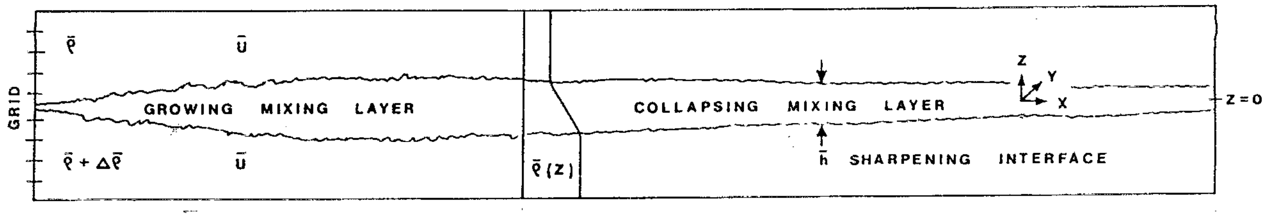


FIG. 1. Schematic of flow configuration and experimental setup.

Lienhard and Van Atta[7] obtained $\epsilon_c/\nu N^2=8.7$.

The purpose of this paper is to show that the data of the present experiments collapse with previous experimental data if scaled appropriately. The collapsed data show that the nondimensional dissipation rate $\epsilon_c/\nu N^2$, as defined in the above-mentioned laboratory experiments is dependent upon Reynolds numbers. We shall argue that the development of the flux F in shear-free, decaying grid-generated turbulence, specifically the initial zero-crossing point of the flux as the flow develops downstream from the grid, is governed by inviscid dynamics. Therefore the use of this criterion for estimating a universal value of $\epsilon_c/\nu N^2$ is inappropriate.

II. EXPERIMENTAL SETUP AND APPARATUS

The details of the closed-circuit two-layer stratified water tunnel (15 cm wide, 20 cm deep, and 225 cm long) and related apparatus that were used in the experiments are described in Huq and Britter.[13] Figure 1 shows the flow configuration: the flow was driven by two pumps and the top water level was maintained by an overflow weir, resulting in a two-layer density stratified, shear-free flow (the mean velocity $\bar{U}=7.7$ cm/s). The density difference between the layers was generated by adding brine (Schmidt number, S_c, the ratio of momentum to species diffusivities, equals 700) to the lower layer reservoir. Coarse and fine screens upstream of the turbulence grid reduced the background turbulence to $w'/\bar{U} = 0.2\%$. The grid comprised bars of width $d=0.64$ and 0.2 cm thickness arranged in square mesh with spacing $M=3.2$ cm between bars (i.e., $M/d=5$, grid solidity=36%). A typical grid Reynolds number was $R_M=\bar{U}M/\nu\sim 2500$. The vertical and streamwise velocity components were determined by quartz-coated TSI cylindrical X films and the density field by an aspirating conductivity probe (spatial resolution about 0.4 mm, frequency response approximately 70 Hz). Flux measurements were obtained by combining the conductivity probe with the X films—the measuring volume of the combined flux probe was estimated to be 1.5 mm. Digital data acquisition and processing was done by a microcomputer. Errors in RMS of the velocity fluctuations and correlation measurements were estimated at typically 5% and 10%, respectively.

III. RESULTS

A. Velocity field

The evolution of the turbulent velocity fluctuations downstream of the grid are shown plotted in Fig. 2. In the unstratified case ($\Delta\bar{\rho}=0$), the data are consistent with a decay law of the form $(\bar{U}/u_i')^2=A[(x-x_0)/M]^J$, has been found by many previous investigators (e.g., Batchelor and Townsend[14] and Stillinger *et al.*[5]). It is apparent from Fig. 2 that stable stratification can affect the vertical velocity fluctuations strongly, while having only a weak effect on the horizontal velocity fluctuation. Our measurements of the flux (discussed in Sec. III B) indicate that the term $(g/\bar{\rho})\overline{\rho w}$, which represents energy extracted from the vertical turbulent kinetic energy in doing work against buoyancy forces, accounts for the changes in the rate of decrease of w'^2, namely $\bar{U}(\partial/\partial x)w'^2$, to within 15%. The dissipation rates therefore remain essentially unchanged. We have determined dissipation rates from $\epsilon \simeq \bar{U}(\partial/\partial x)(\frac{1}{2}\overline{q^2})+(g/\bar{\rho})\overline{\rho w}$ using the above decay law fitted to the data to estimate streamwise derivatives $\bar{U}(\partial/\partial x)\frac{1}{2}\overline{q^2}$. We found $\epsilon/\bar{U}^3/M =C(x/M-x_0/M)^{-2}$ to within 10% for all cases with $x_0/M=5$ and $C=0.027$. In the above the turbulent kinetic energy $\frac{1}{2}\overline{q^2}$ is defined to be $\frac{1}{2}(2u'^2+w'^2)$, as the v' component is not measured. As shown in Fig. 3, the similarity of the forms of the decay of the turbulent kinetic energy for the cases with density stratification and in the absence of stratification ($\Delta\rho=0$) indicate similar rates of dissipation.

B. Fluxes

The effect of stable stratification on the normalized buoyancy flux $F = \overline{\rho w}/\rho' w'$ is shown in Fig. 4(a). The measurements show that close to the grid F approaches the value of 0.5 for $\Delta\bar{\rho} = 0.3$ kg/m^3 as for the case without stratification ($\Delta\bar{\rho} = 0$). (For $\Delta\bar{\rho}=0$, F is a scalar flux of marked fluid. Details of the passive scalar case are given in Huq and Britter[13]). For larger density differences the peak value is smaller and the decay is faster, with significant negative values (i.e., countergradient flux) and with oscillatory evolution for large downstream distances.

We have found, in agreement with Jayesh and Warhaft,[9] that the buoyancy frequency N at the center of the interface in these experiments is independent of x/M for a given density difference (see Fig. 5). This facilitates the use of the parameter Nt, namely, the ratio of the turbulent time scale $t=l/u'$ to the buoyancy time scale N^{-1} to characterize the evolution of the flow. The turbulence time scale is proportional to $x/\bar{U}$, the advection time from the grid, in these experiments. The flux is plotted as a function of $Nx/\bar{U}$ in Fig. 4(b), and the data from all our experiments can be seen to be collapsed.

Peak values of $F\simeq 0.45$ occur at $Nt\simeq 1$ (values similar to Itsweire *et al.*[6]), $F\simeq 0$ at $Nt\simeq 4$, the maximum value ($\simeq -0.1$) of the countergradient flux occurring at $Nt\simeq 5$, and

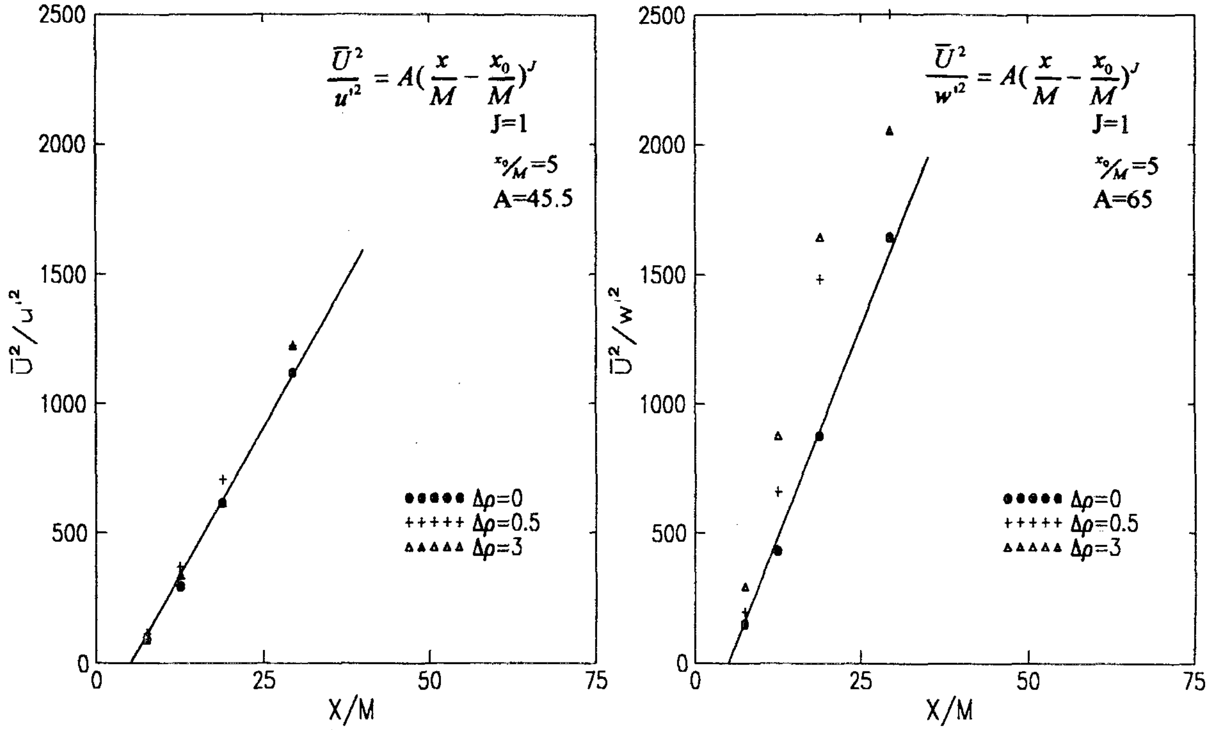


FIG. 2. Evolution of turbulent velocity fluctuations at the center of the interface for various values of density difference. Full lines are decay curves for the unstratified case ($\Delta\bar{\rho} = 0$) for u' and w'. Equations for full lines are given in the figure.

$F \simeq 0$ again at $Nt \simeq 6.5$, with small oscillating values of F thereafter. The evolution and the values of F are similar to the results from wind tunnel experiments of Jayesh *et al.*,[9] with the identical configuration of two homogeneous layers of differing densities coflowing without shear. They found a maximum value of $F \simeq 0.65$ as opposed to the present value of 0.5. This disparity is consistent with observations of Turner,[2] who noted that scalar fluxes in cases of heat-induced buoyancy were greater than in cases for salt-induced buoyancy. Thus, flux in heated wind tunnels where the $S_c = 0.7$ may be expected to be greater than the present water tunnel ($S_c = 700$) values.

C. Dissipation rates when $\overline{\rho w} \rightarrow 0$

The first zero-crossing point of the flux curves of Fig. 4(a) were used to define a dissipation rate ϵ_0 for $\Delta\bar{\rho} = 3,2,1,$ and $0.5\ \mathrm{kg/m^3}$ for which the values of N at the center of the interface were $N = 0.98$, 0.8, 0.47, and $0.38\ \mathrm{s^{-1}}$, respectively. These dissipation rates, nondimensionalized by νN^2, are plotted against the Froude number $NM/\bar{U}$ in Fig. 6. Note that we make a distinction between ϵ_0, the dissipation rate at the initial location where $\overline{\rho w} = 0$, and ϵ_c. Also shown in Fig. 6 are the nondimensional dissipation rate $\epsilon_0/\nu N^2$ from other experiments in which flux $\overline{\rho w}$ was measured (Stillinger *et al.*,[5] Itsweire *et al.*,[6] Lienhard and Van Atta,[7] Yoon and Warhaft,[8] and Jayesh *et al.*[9]). These experiments include different configurations (stepwise and linear stratification), different values of Schmidt number ($S_c = 0.7$ and 700 for wind and water tunnel experiments), and grid Reynolds numbers, $2500 \leqslant R_M \leqslant 10\,000$. Recall

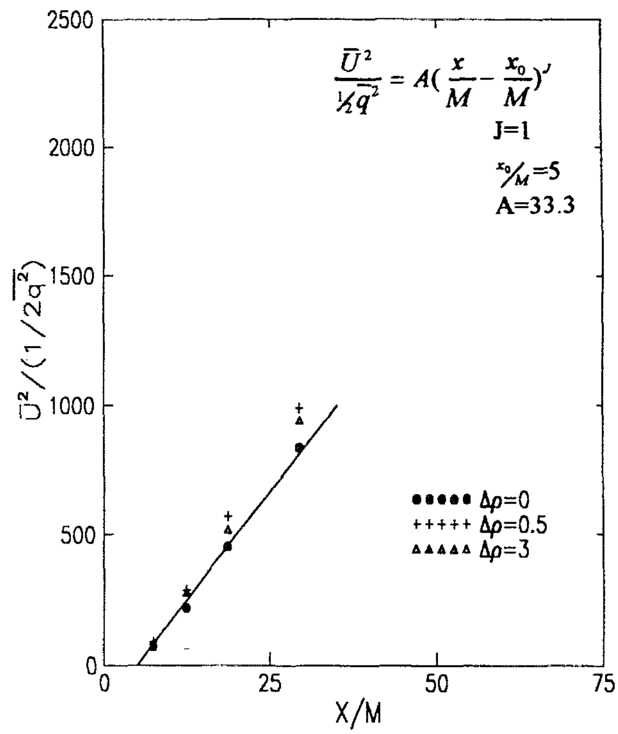
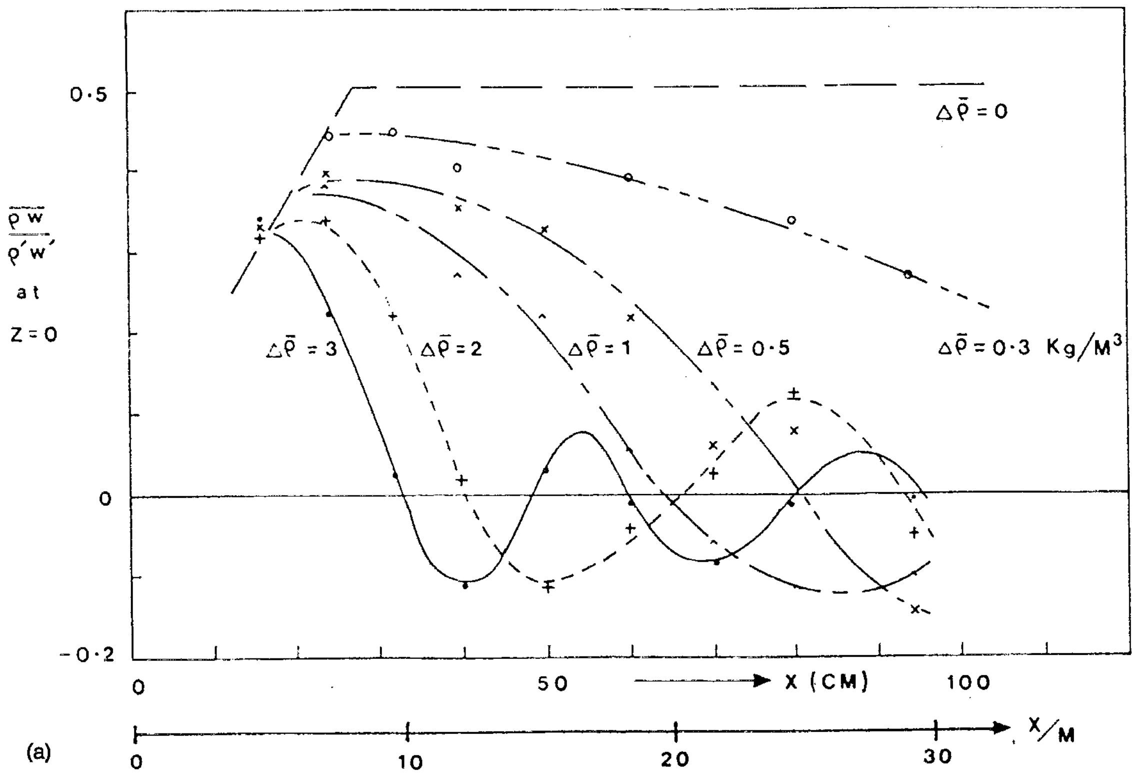


FIG. 3. Evolution of turbulent kinetic energy $\frac{1}{2}\overline{q^2}$ at the center of the interface for various values of density difference. The full line is the decay curve for the unstratified case ($\Delta\bar{\rho} = 0$) for $\frac{1}{2}\overline{q^2}$. The equation for a full line is given in the figure.



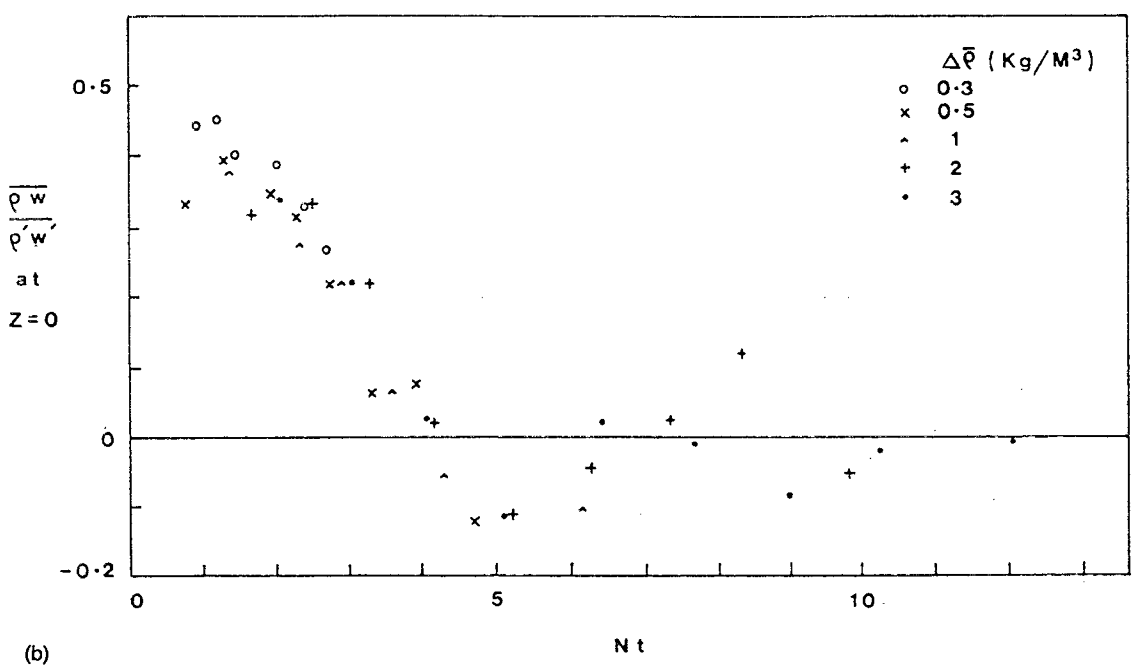


FIG. 4. Evolution of the normalized buoyancy flux at the center of the interface. (a) For various density differences; (b) Nt scaling.

that Fig. 4(a) shows that the location of the initial zero-crossing point of the flux depends on the density difference (i.e., $\Delta\bar{\rho}/\bar{\rho}$ or N). Also recall that the dissipation rate ϵ for decaying grid-generated turbulence varies with distance from the grid, that is, ϵ varies with the advection time $x/\bar{U}$ from the grid (i.e., the age of the flow). There is considerable scatter in the data, however, an increasing trend in the values of $\epsilon_0/\nu N^2$ with R_M is apparent, with no obvious dependence on $NM/\bar{U}$.

A prediction or consistency check on the scaling of the nondimensional dissipation rate $\epsilon_0/\nu N^2$ for shear-free, decaying grid-generated stratified turbulence is obtained by considering the evolution of the kinetic energy of the turbulence. Following Tennekes and Lumley[15] for neutrally stratified, shear-free, decaying grid-generated turbulence, the evolution of the turbulent kinetic energy $\frac{1}{2}\overline{q^2}$ is approximately (also recall Fig. 3)

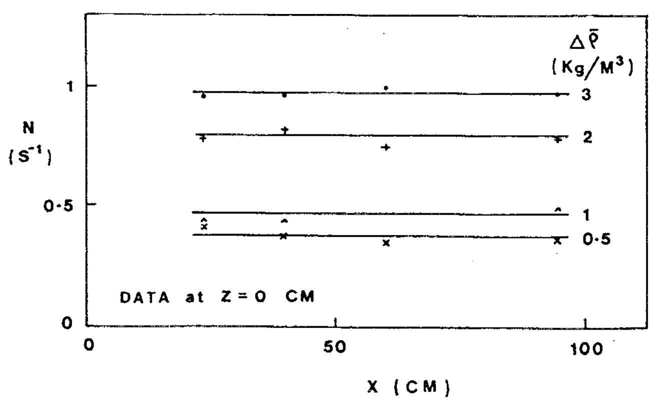


FIG. 5. Buoyancy frequency N at the center of the interface for various values of density difference.

$$\tfrac{1}{2}\overline{q^2}/\bar{U}^2 \sim (x/M)^{-1}. \tag{1}$$

Optimal values of the exponent may be slightly different from one (Comte-Bellot and Corrsin[16]), but this does not affect our argument below. The kinetic energy equation is

$$\frac{1}{2}\bar{U}\frac{\partial}{\partial x}\overline{q^2} = -\epsilon. \tag{2}$$

A decay time scale T_L of the eddies may be defined as

$$T_L = \frac{1/2\overline{q^2}}{\epsilon}, \tag{3}$$

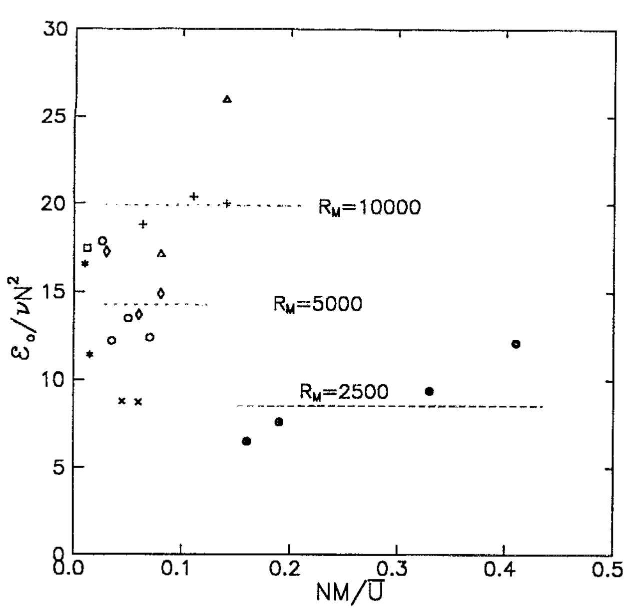


FIG. 6. Plot of nondimensional dissipation rate $\epsilon_0/\nu N^2$ at $\overline{\rho w}=0$ against the Froude number $MN/\bar{U}$ for present and previous shear-free, decaying grid-generated turbulence experiments. Symbols: ◇, Stillinger *et al.* (1983); △, Run 36, Itsweire *et al.*;[6] +, Run 37, Itsweire *et al.*;[6] ○, Run 52, Itsweire *et al.*;[6] ×, Lienhard and Van Atta;[7] □, Yoon and Warhaft;[8] *, Jayesh *et al.*;[9] ●, present data. Dashed line (---) shows trend of the present data. Dot–dashed lines show approximate trend of other experimental data.

where $T_L = x/\bar{U}$ from (1)–(3). For stratified turbulence with uniform density stratification, similar arguments may be applied to the total turbulent energy E comprising TKE/unit mass and TPE/unit mass, where TKE/unit mass $(=\frac{1}{2}u_i'^2)$ and TPE/unit mass $[=-\frac{1}{2}(g/\bar{\rho})(\partial\bar{\rho}/\partial z)^{-1}\overline{\rho^2}]$. Therefore

$$T_L' = \frac{E}{\epsilon_{\mathrm{TKE}}+\epsilon_{\mathrm{TPE}}}, \tag{4}$$

where ϵ_{TKE} and ϵ_{TPE} are the dissipation rates for turbulent kinetic and potential energy, defined as $\epsilon_{\mathrm{TKE}}=\nu\overline{(\partial u_i/\partial x_j)^2}$ and $\epsilon_{\mathrm{TPE}} = -\kappa g/\bar{\rho}(\partial\bar{\rho}/\partial z)^{-1}\overline{(\partial\rho/\partial x_j)^2}$. If $E \sim x^{-1}$, as suggested by observations and experiments (Riley[17] and Stretch[18]), then $T_L' \sim x/\bar{U}$. Note that the time scale of the most energetic eddies at each value of x is of the order of the travel time, $x/\bar{U}$, from the grid. This indicates that for shear-free, decaying grid-generated turbulence the large eddies "remember" the initial condition at the grid.

Based on our observations of the flux scaling with Nt [see Fig. 4(b)] and on previous analysis using inviscid Rapid Distortion Theory (Hunt *et al.*[19]), we hypothesize that independent of Reynolds number (assumed sufficiently large), Nt=const when $\overline{\rho w}\rightarrow 0$ in these experiments. Then $Nx_0/\bar{U}$=const and so

$$x_0 \sim \bar{U}/N. \tag{5}$$

The initial zero-crossing point is then assumed to scale with N and to be independent of the kinematic viscosity ν. If $\frac{1}{2}q^2/\bar{U}^2 \sim (x/M)^{-1}$ and also $E \sim x^{-1}$, then, using (1) and (2),

$$\bar{U}\frac{\partial}{\partial x}E \sim \frac{\bar{U}^3}{M}\left(\frac{x}{M}\right)^{-2}, \tag{6a}$$

that is,

$$\epsilon \sim \frac{\bar{U}^3}{M}\left(\frac{x}{M}\right)^{-2}. \tag{6b}$$

Now, using Eqs. (5) and (6b) yields

$$\epsilon_0 \sim \frac{\bar{U}^3}{M}\left(\frac{MN}{\bar{U}}\right)^2, \tag{7}$$

so that our hypothesis predicts

$$\frac{\epsilon_0}{\bar{U}^3/M} \sim \left(\frac{MN}{\bar{U}}\right)^2. \tag{8}$$

The available experimental data are presented in Fig. 7 in the nondimensional forms required to check the validity of Eq. (8). The agreement is good. For all these experiments of shear-free, decaying grid-generated turbulence, nondimensional dissipation rates (where $\overline{\rho w}$ initially vanishes) can be collapsed in a manner consistent with Eq. (8). Note that the data from these experiments have been collapsed using totally inviscid dimensional analysis. It is further noted that Eq. (8) implies that $\epsilon_0/\nu N^2 \propto uM/\nu$, which is consistent with the trend previously noted in Fig. 6.

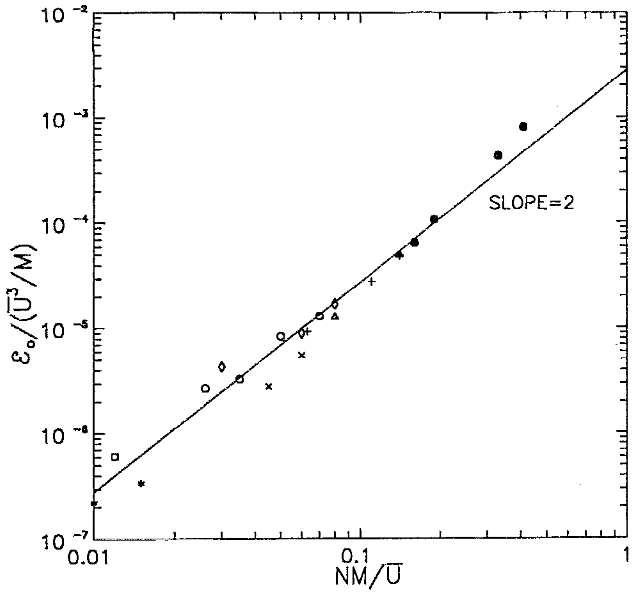


FIG. 7. Plot of nondimensional dissipation rate $\epsilon_0/(\bar{U}^3/M)$ against Froude number $NM/\bar{U}$. Full line (—) shows +2 power law dependence of $\epsilon_0/(\bar{U}^3/M)$ against $NM/\bar{U}$. Symbols are as in Fig. 6.

IV. CONCLUSION

We have shown that the dissipation rate from a variety of laboratory experiments on shear-free, decaying grid-generated turbulence, at the point where the flux initially goes to zero, can be scaled as $\epsilon_0/(\bar{U}^3/M)\sim(MN/\bar{U})^2$. This has been deduced from purely inviscid scaling arguments and based on the observation that $F\rightarrow 0$ at a constant value of $Nx_0/\bar{U}$. The results indicate that the quantity $\epsilon_0/\nu N^2$ inferred at the initial zero-crossing point of the flux curve is proportional to the Reynolds number $R=\bar{U}M/\nu$. These results illustrate that the initial zero-crossing point of the flux curve in these developing flows is an inappropriate criterion for determining a universal value of the critical dissipation rate. It is our view that ϵ_0 (based on the point where flux goes to zero initially in these experiments) does not represent any "critical condition" for these transient (or developing flow) experiments. Rather, it represents a point in the temporal evolution of the flow, as determined by the initial conditions at the grid, the advection time $x/\bar{U}$, and inviscid buoyancy-inertial dynamics.

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